Synopsis

There is a growing consensus among physicists that the classical notion of spacetime has to be drastically revised in order to find a consistent formulation of quantum mechanics and gravity. One such nontrivial attempt comprises of replacing functions of continuous spacetime coordinates with functions over noncommutative algebra. Dynamics on such noncommutative spacetimes (noncommutative theories) are of great interest for a variety of reasons among the physicists. Additionally arguments combining quantum uncertainties with classical gravity provide an alternative motivation for their study, and it is hoped that these theories can provide a self-consistent deformation of ordinary quantum field theories at small distances, yielding non-locality, or create a framework for finite truncation of quantum field theories while preserving symmetries.

In this thesis we study the gauge theories on noncommutative Moyal space. We find new static solitons and instantons in terms of the so-called generalized Bose operators (GBO). GBOs are constructed to describe reducible representation of the oscillator algebra. They create/annihilate k-quanta, k being a positive integer. We start with giving an alternative description to the already found static magnetic flux tube solutions of the noncommutative gauge theories in terms of GBOs. The Nielsen-Olesen vortex solutions found in terms of these operators also reduce to the ones known in the literature. On the other hand, we find a class of new instanton solutions which are unitarily inequivalent to the ones found from ADHM construction on noncommutative space. The charge of the instanton has a description in terms of the index representing the reducibility of the Fock space representation, i.e., k. After studying the static soliton solutions in noncommutative Minkowski space and the instanton solutions in noncommutative Euclidean space

we go on to study the implications of the time-space noncommutativity in Minkowski space. To understand it properly we study the time-dependent transitions of a forced harmonic oscillator in noncommutative 1+1 dimensional spacetime. We also provide an interpretation of our results in the context of non-linear quantum optics. We then shift to the so-called DSR theories which are related to a different kind of noncommutative (κ-Minkowski) space. DSR (Doubly/Deformed Special Relativity) aims to search for an alternate relativistic theory which keeps a length/energy scale (the Planck scale) and a velocity scale (the speed of light scale) invariant. We study thermodynamics of an ideal gas in such a scenario.

In first chapter we introduce the subjects of the noncommutative quantum theories and the DSR. Chapter 2 starts with describing the GBOs. They correspond to reducible representations of the harmonic oscillator algebra. We demonstrate their relevance in the construction of topologically non-trivial solutions in noncommutative gauge theories, focusing our attention to flux tubes, vortices, and instantons. Our method provides a simple new relation between the topological charge and the number of times the basic irreducible representation occurs in the reducible representation underlying the GBO. When used in conjunction with the noncommutative ADHM construction, we find that these new instantons are in general not unitarily equivalent to the ones currently known in literature.

Chapter 3 studies the time dependent transitions of quantum forced harmonic oscillator (QFHO) in noncommutative $\mathbb{R}^{1,1}$ perturbatively to linear order in the noncommutativity θ. We show that the Poisson distribution gets modified, and that the vacuum state evolves into a "squeezed" state rather than a coherent state. The time evolutions of uncertainties in position and momentum in vacuum are also studied and imply interesting consequences for modelling nonlinear phenomena in quantum optics.

In chapter 4 we study thermodynamics of an ideal gas in Doubly Special Relativity. We obtain a series solution for the partition function and derive thermodynamic quantities. We observe that DSR thermodynamics is non-perturbative in the SR and massless limits. A stiffer equation of state is found. We conclude our results in the last chapter.